Chapter 3
Elementary Algebra

Summary This short chapter is a brief statement, outlining our most familiar algebra and its unavoidable importance to Quantum Mathematics.

3.1 Elementary Algebra Exerts Impact on Quantum Mechanics

It is surely ironic that, rather than any one of the sophisticated abstract algebras which have grown out of Quantum Theory, quantum indeterminacy rests hidden within *that* algebra which has stood before us all along: the algebra we learnt at school, the ubiquitous algebra that lies beneath Applied Mathematics, the algebra upon which Quantum Mathematics foundationally sits — the algebra of scalars — *Elementary Algebra.*

Elementary Algebra is taken for granted throughout Applied Mathematics, to the extent that, we never question any inherent restriction it might have on Physical Theory. For indeed; within Elementary Algebra, whilst *proof* and *consequence* are well understood, for the physicist there is unrealised: *consistence, process, computability, accidental coincidence* and *stability*; all to be explored.

This perspective on Elementary Algebra is not proposed as a way of doing Quantum Mechanics; it is a mathematics of *existence* whose purpose is insight. It is an approach to Mathematical Physics, which, in addition to physical effects that manifest through *cause*, effects which manifest through *non-prevention,* are represented also

3.2 What is Elementary Algebra?

Elementary Algebra is the abstraction of Elementary Arithmetic. It embodies the traditional operations of addition and multiplication, along with their reciprocal operations: subtraction and division.

When all four of these operations are accepted as rules obeyed by numbers, *algebraic closure* demands existence of all *scalars* we commonly use in Applied Mathematics. These include all the rational numbers $\mathbb{Q}$, all the numbers on the real line $\mathbb{R}$, and all those on the complex plane $\mathbb{C}$. Elementary Algebra is not the algebra for integers $\mathbb{N}$, because $\mathbb{N}$ is not closed under the division operation.

An important point to notice is that the complex numbers are often depicted as 2-dimensional *vectors* in $\mathbb{R}^2$ (for understandable reasons) when rightly they are *numbers* or *scalars* in $\mathbb{C}^1$.

3.3 As Formal System

A formal system comprises: a precise language, rules for writing formulae, and further rules of deduction. In order to see the full scope of Elementary Algebra's capabilities, it shall be treated as a *formal axiomatised system* in context of Mathematical Logic. For this, the four operations are embodied in the *Field Axioms* given in table 8.1, page 52. This formalised Elementary Algebra comprises all statements written in the language of, and operating under the rules of the *Field Axioms*. The structures $\mathbb{Q}$, $\mathbb{R}$, and $\mathbb{C}$ are infinite fields of scalars. These are structures that satisfy the *Field Axioms*. There may also be other, more obscure fields, satisfying the Field Axioms, which are less obvious.

3.4 The Inherent Logic

As opposed to the classical logic of *true* and *false*, when Elementary Algebra is treated as a *formal system*, any formula written in the language of Elementary Algebra is either *provable*, *disprovable* or otherwise, *logically independent* of the Field Axioms. In particular, within Elementary Algebra, any proposition asserting existence of the imaginary unit, explicitly or implicitly, is logically independent of the Field Axioms. This is in contrast to all *rationals*, which are *logically dependent* on Axioms. Together, this *independence* and *dependence* furnish a logic, comprising values: *provable*, *negatable* and *'neither provable nor negatable'* — logic comparable to Reichenbach's.