Chapter 9
Pure State Vienna Experiments are Non-Unitary

Summary This chapter derives and discovers algebra demanded by the Boolean system, used in the Vienna Experiments, unexpected and overlooked by the Vienna Team. That algebra contradicts any *a priori* or axiomatic *unitary|Hermitian Postulate*: by demanding freedom from unitarity for pure states — but allowing newly ingressed, logically independent unitarity, in the creation of mixed states.

9.1 Vienna Experiments are not Isomorphic with Pauli

Readers of chapter 6, or the Paterek paper [19], are likely to infer that the Vienna Team's mappings (6.2), from Pauli operators to Boolean pairs, imply a *one–one* and *onto* (bijective) correspondence linking the Pauli operators with Boolean pairs. I quote here an extract from their conclusionary remarks [19]:

> In conclusion, we have demonstrated that the dependence or independence of certain mathematical propositions in a finite axiomatic set can be tested by performing corresponding Pauli group measurements. (It would be interesting to investigate the possibility of extending our results beyond this class of measurements.) This is achieved via an isomorphism between axioms and quantum states as well as between propositions and quantum measurements. Dependence (independence) is revealed by definite (random) outcomes. Having this isomorphism, logical independence need not be proved by logic but can be inferred from experimental results. From the foundational point of view, this sheds new light on the (mathematical) origin of quantum randomness in these measurements. ...

The actual picture is *one–many*. Implication is directed only *from* the Pauli products, to the Boolean pairs, in the sense of the arrows shown here:

$$\sigma_{\mathsf{z}} = \sigma_{\mathsf{x}}^0 \sigma_{\mathsf{z}}^1 \to (0,1) \qquad \sigma_{\mathsf{x}} = \sigma_{\mathsf{x}}^1 \sigma_{\mathsf{z}}^0 \to (1,0) \qquad -i\sigma_{\mathsf{y}} = \sigma_{\mathsf{x}}^1 \sigma_{\mathsf{z}}^1 \to (1,1) \qquad (9.1)$$

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Written less formally, formulae asserted by the Boolean pairs are these:

$$\mathbb{1} = \mathsf{a}^0\mathsf{c}^0 \qquad \mathsf{c} = \mathsf{a}^0\mathsf{c}^1 \qquad \mathsf{a} = \mathsf{a}^1\mathsf{c}^0 \qquad \eta^{-1}\mathsf{b} = \mathsf{a}^1\mathsf{c}^1 \tag{9.5}$$

The first of these was not mentioned upto this point. It corresponds to the Boolean pair (0,0), which is never invoked in the forward sense of (9.1); but in the backward sense, it is prudent to acknowledge its possibility, and I prefer it be not ignored.

9.2 Information Content of the Pauli Algebra

An algebra can be considered to be a composite of all logically independent items of information that derive it. And so, it is instructive to compare the content of (9.5) against a list of six statements, from which we may derive the Pauli algebraic statement:

$$-i\mathsf{b} = \mathsf{ac} \tag{9.6}$$

That list consists of the six statements (9.12)–(9.17). Starting from this list, the derivation procedure that I give is an adaption of a proof shown by W E Baylis, J Huschilt and Jiansu Wei [3]; although it likely originates with David Hestenes [10]. The purpose of writing out this derivation is to illustrate the kind of logically independent statements which can be expected, that make up (9.6); but I must admit beforehand, that this derivation is a bit misleading, because actually, there are proofs involving fewer of the six statements. The actual proof of interest, I give afterwards, in section 9.3.

Remark 10. It would seem that only three out of the six, plus closure over products, are needed for a proof; and that different selections of the three are capable of doing the job.

The Pauli algebra is a Lie algebra; and hence, is a linear vector space. Accordingly, I begin with information inherited from the vector space axioms, and then add other information peculiar to the Pauli Lie algebra, $\mathsf{su}(2)$.

Closure: For any two vectors u and v, there exists a vector w such that

$$\mathsf{w} = \mathsf{u} + \mathsf{v}$$

Identities: There exist additive and multiplicative identities, $\mathbb{0}$ and $\mathbb{1}$, such that, for any arbitrary vector v:

$$\mathsf{v}\mathbb{1} = \mathbb{1}\mathsf{v} = \mathsf{v} \tag{9.7}$$

$$\mathsf{v} + \mathbb{0} = \mathbb{0} + \mathsf{v} = \mathsf{v} \tag{9.8}$$

$$\mathsf{v}\mathbb{0} = \mathbb{0}\mathsf{v} = \mathbb{0} \tag{9.9}$$

Additive inverse: For any arbitrary vector v, there exists an additive inverse $(-\mathsf{v})$ such that

$$(-\mathsf{v}) + \mathsf{v} = \mathbb{0} \tag{9.10}$$

Scaling: For any arbitrary vector v, and any scalar a, there exists a vector u such that

$$\mathsf{u} = a\mathsf{v} \tag{9.11}$$

Products: A feature of Lie algebras is that its arbitrary vectors, u and v, are linear operators; and hence there exist products uv and vu. Closure within the Lie algebra of these simple products is not a requirement; they are not necessarily members of the Lie algebra.

Closure over Commutators: Commutators of these products are members of the Lie algebra vector space.

Dimension: Assume a 3 dimensional vector space, with independent basis: a, b, c.

The list of six statements

Involutory information: Assume all three basis vectors are involutory:

$$\mathsf{aa} = \mathbb{1} \qquad \mathsf{a}\text{ involutory} \tag{9.12}$$

$$\mathsf{bb} = \mathbb{1} \qquad \mathsf{b}\text{ involutory} \tag{9.13}$$

$$\mathsf{cc} = \mathbb{1} \qquad \mathsf{c}\text{ involutory} \tag{9.14}$$

Orthogonal information: Assume products between basis vectors are orthogonal:

$$\mathsf{ab} + \mathsf{ba} = \mathbb{0} \qquad \mathsf{ab}\text{ orthogonal} \tag{9.15}$$

$$\mathsf{bc} + \mathsf{cb} = \mathbb{0} \qquad \mathsf{bc}\text{ orthogonal} \tag{9.16}$$

$$\mathsf{ca} + \mathsf{ac} = \mathbb{0} \qquad \mathsf{ca}\text{ orthogonal} \tag{9.17}$$

9.4 New Insight

The Vienna Experiments into photon polarisation invoke isomorphism, not with the Pauli algebra itself, but in general, with a broader algebra which asserts only *involutarity* — absent of orthogonality. However, orthogonality is free to enter this involutory algebra as extra newly ingressed information; and under circumstances where it does, this broader algebra then restricts to the (unitary) Pauli algebra.

So what are those circumstances? Referring back to Figure 6.1; during the evolution of the parallel (predictable) experiment, comparison of the input and output propositions indicates no change; whereas the same comparison for the orthogonal experiments does show an ingress of independent information. That means, in mixed state experiments there is ingression of new information.

In chapter 11 we shall see that that new information originates in commutator processes involving orthogonal operators, going on in the density matrix, in mixed state experiments. But in parallel experiments, commutator processes involving orthogonality, never occur. That means orthogonal operators are needed in representing mixed state processes, but are not needed for pure states.

This confirms that the Vienna Experiments represent pure states using non-unitary, non-Pauli algebra. And further that, the *absence* or *presence* of orthogonality in representation is a predictor of *predictability* or *randomness*, respectively.

There is insight to be gained about indeterminacy's *indefiniteness*, by recognising that the Pauli algebra is *referentially ambiguous* because no statement in the algebra can specify the left|right handedness of the Pauli algebra. This is ambiguity which can be eliminated only from outside the algebra — outside in the same sense as 'outside the system' when talking about Gödel's Incompleteness theorems. In contrast, in the pure state algebra, there is no such ambiguity in left|right handedness.