Chapter 11
The Inaccessible Lost History at Measurement

Summary This chapter closely examines density operator processes going on in mixed state Vienna Experiments. The purpose is to properly understand what Boolean information of chapter 6 means, in terms of Pauli algebraic processes. Doing this reveals how indeterminacy's *indefiniteness* has it roots in *referential ambiguity.* Note: Whereas chapters 9 and 10 treat unitarity as *logically independent* information which is either, in presence, or in absence; this chapter treats unitarity as always present, but *possible*|*necessary*, in the sense of Modal Logic.

11.1 Involvement of the Density Operator

At the heart of the Vienna Team's methodology is the *density matrix.* This conveys photon polarisation information, through experiments. Mathematically speaking, we think of the density matrix as a matrix mechanical operator, written in terms of matrix transformations. Yet, the data that was extracted from those experiments is written in terms of simple input|output Boolean information. The intricacies of just how that Boolean information communicates with the density matrix is crucial to understanding indeterminacy's indefiniteness.

The density operator has a working anatomy consisting of a Pauli product part, and a part comprising the *orthogonality index* $\mathcal{N}_{\mathsf{B}}$. See section 6.4.1. As the density matrix evolves through the experiment, the Pauli products generate increments in the Boolean information, and these accumulate in the orthogonality index. Then, at the point where the experiment asks for a measurement, this process goes into reverse. But that reverse process is not so simple. It is these processes that this chapter scrutinises.

11.2 Referential Ambiguity in the Density Matrix

As said in section 2.6, to reveal the logic inherent in Quantum Mathematics, the *whole information* of the quantum system must be taken into account. Critical in this respect, unlike probability density, the density matrix conveys probability information about the full set of all complimentary states. But that itself is not enough; it is the following radical approach that enables logical independence in the Vienna Experiments to be revealed.

The unique form of density operator used in the Vienna Experiments is wondrously ingenious: exposing the machinery of information conservation, and loss. The orthogonality index behaves as memory, registering *absolute* (Bloch) orientations of polarisers, as set by the experiment hardware configuration. As the density matrix evolves, those absolute orientations, originally subtended by polarisers and then conveyed by photons, resolve to a *relative* (Bloch) orthogonality. Quite wonderfully, the memory performance of the orthogonality index is *faithful* or *unfaithful*, dependent on that relative orthogonality. For pure states, the original absolute orientations of polarisers are faithfully remembered. For mixed states, only their relative orthogonality is remembered; the absolute orientations are lost.

This is memory behaviour going on in interactions between the orthogonality index and Pauli products. In a little more detail. As the density operator evolves, the orthogonality index receives increments generated by commutators in Pauli algebraic processes. Experiments whose orthogonality index becomes *set,* characterises photons prepared as mixed states. These have ambiguous history because their orthogonality index has no memory of how the orthogonality value was generated — whether for instance, it originated through the product-sequence $\sigma_x\sigma_z\sigma_x^\dagger$, or the product-sequence $\sigma_y\sigma_z\sigma_y^\dagger$. In other words, the actual historical polarisation is confused with its (Bloch) reflection. This presents a referential ambiguity. In experiments involving polarisers aligned (Bloch) orthogonally, measurements cannot access that historical information, unambiguously.

On the other hand, experiments prepared as pure states, whose orthogonality index is *not set,* suffer no ambiguous history.

11.3 Algebraic Processes in the Density Matrix

Recall from section 6.4 that Blackbox-Processes take polarisation orientations as input; these then *determine* the density matrix downstream of the Blackbox. Measurement-Processes then attempt to extract that polarisation information from that density matrix. That is to say: the Blackbox writes definite values $\overset{\square}{f}(0)$ and $\overset{\square}{f}(1)$ to the density matrix; then Measurement-Processes attempt to read them from it. But in the case of mixed states, from the standpoint of the Measurement-Processes the definite polarisation orientations are ambiguously confused with their (Bloch) reflections. As a result, values that Measurement-Processes actually read are $\overset{\diamond}{f}(0)$ and $\overset{\diamond}{f}(1)$; and these are determinate or indeterminate, according to whether the polarisation state is pure or mixed.

In sections that follow, these Blackbox-Processes and Measurement-Processes are re-examined; the propose being, to understand the meaning of the Boolean processes in terms of transformations and symmetries. Doing that exposes algebraic manipulations that involve swapping the sequence-order of operator-products, resulting in the generation of commutators; followed by un-swapping, resulting in commutator re-absorption — an overall irreversible process.

Section 11.4 examines the Blackbox-Processes, the formulae (11.1)→(11.3), where commutator values accumulate in the orthogonality index $\mathscr{N}_{\mathsf{B}}$. Section 11.5 examines the Measurement-Processes, the formulae (11.4)→(11.6); these *attempt* exactly the reverse or 'undoing' of the Blackbox-Processes. They attempt to 'un-swap' operators, so as to 'undo' or re-absorb the commutators.

The overall process (11.1)→(11.6) is the conversion of (orthogonal) *geometric* information into *scalar* information; followed then by the reversal of that. This reversal results in referential ambiguity, because the geometry–scalar relationship is not *one–one* and *onto*.

11.4 Density Operator Generation is Deterministic

The Blackbox, takes $\overset{\square}{f}(0)$ and $\overset{\square}{f}(1)$ as input, then algebraic processes determine a definite value for the orthogonality index $\mathscr{N}_{\mathsf{B}}$, and hence, a definite density matrix ρ_{B}, at (11.3). The underbracketing shown in the lead-up to (11.3) is intended to point out where the action is taking place.

$$
\begin{aligned}
\rho_{\mathsf{B}} = U_{\mathsf{B}}\rho_{\mathsf{P}}\,U_{\mathsf{B}}^{\dagger} &= \frac{1}{2}\left[\mathbb{1} + \lambda_{mn} i^{mn}\left[\sigma_{\mathsf{x}}^{\overset{\square}{f}(0)}\sigma_{\mathsf{z}}^{\overset{\square}{f}(1)}\right]\left[\sigma_{\mathsf{x}}^{m}\sigma_{\mathsf{z}}^{n}\right]\left[\sigma_{\mathsf{x}}^{\overset{\square}{f}(0)}\sigma_{\mathsf{z}}^{\overset{\square}{f}(1)}\right]^{\dagger}\right] \qquad (11.1)\\
&= \frac{1}{2}\left[\mathbb{1} + \lambda_{mn} i^{mn}\sigma_{\mathsf{x}}^{\overset{\square}{f}(0)}\sigma_{\mathsf{z}}^{\overset{\square}{f}(1)}\sigma_{\mathsf{x}}^{m}\,\underbrace{\sigma_{\mathsf{z}}^{n}\sigma_{\mathsf{z}}^{\overset{\square}{f}(1)}}\,\sigma_{\mathsf{x}}^{\overset{\square}{f}(0)}\right]\\
&= \frac{1}{2}\left[\mathbb{1} + \lambda_{mn} i^{mn}\sigma_{\mathsf{x}}^{\overset{\square}{f}(0)}\sigma_{\mathsf{z}}^{\overset{\square}{f}(1)}\sigma_{\mathsf{x}}^{m}\,\underbrace{\sigma_{\mathsf{z}}^{\overset{\square}{f}(1)}\sigma_{\mathsf{z}}^{n}}\,\sigma_{\mathsf{x}}^{\overset{\square}{f}(0)}\right]\\
&= \frac{1}{2}\left[\mathbb{1} + \lambda_{mn} i^{mn}\sigma_{\mathsf{x}}^{\overset{\square}{f}(0)}\sigma_{\mathsf{z}}^{\overset{\square}{f}(1)}\,\underbrace{\sigma_{\mathsf{x}}^{m}\sigma_{\mathsf{z}}^{\overset{\square}{f}(1)}}\,\sigma_{\mathsf{z}}^{n}\sigma_{\mathsf{x}}^{\overset{\square}{f}(0)}\right]\\
&= \frac{1}{2}\left[\mathbb{1} + \lambda_{mn}\,\underbrace{(-1)^{m\overset{\square}{f}(1)}}\,i^{mn}\sigma_{\mathsf{x}}^{\overset{\square}{f}(0)}\sigma_{\mathsf{z}}^{\overset{\square}{f}(1)}\,\underbrace{\sigma_{\mathsf{z}}^{\overset{\square}{f}(1)}\sigma_{\mathsf{x}}^{m}}\,\sigma_{\mathsf{z}}^{n}\sigma_{\mathsf{x}}^{\overset{\square}{f}(0)}\right]
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{2}\left[\mathbb{1} + \lambda_{mn}\,(-1)^{m\overset{\square}{f}(1)}\, i^{mn}\sigma_{\mathsf{x}}^{\overset{\square}{f}(0)}\,\underbrace{\sigma_{\mathsf{z}}^{\overset{\square}{f}(1)}\sigma_{\mathsf{z}}^{\overset{\square}{f}(1)}}\,\sigma_{\mathsf{x}}^{m}\sigma_{\mathsf{z}}^{n}\sigma_{\mathsf{x}}^{\overset{\square}{f}(0)}\right]\\
&= \frac{1}{2}\left[\mathbb{1} + \lambda_{mn}\,(-1)^{m\overset{\square}{f}(1)}\, i^{mn}\sigma_{\mathsf{x}}^{\overset{\square}{f}(0)}\,\underbrace{\mathbb{1}}\,\sigma_{\mathsf{x}}^{m}\sigma_{\mathsf{z}}^{n}\sigma_{\mathsf{x}}^{\overset{\square}{f}(0)}\right]\\
&= \frac{1}{2}\left[\mathbb{1} + \lambda_{mn}\,(-1)^{m\overset{\square}{f}(1)}\, i^{mn}\sigma_{\mathsf{x}}^{\overset{\square}{f}(0)}\sigma_{\mathsf{x}}^{m}\,\underbrace{\sigma_{\mathsf{z}}^{n}\sigma_{\mathsf{x}}^{\overset{\square}{f}(0)}}\right]\\
&= \frac{1}{2}\left[\mathbb{1} + \lambda_{mn}\,(-1)^{m\overset{\square}{f}(1)}\,\underbrace{(-1)^{n\overset{\square}{f}(0)}}\, i^{mn}\sigma_{\mathsf{x}}^{\overset{\square}{f}(0)}\sigma_{\mathsf{x}}^{m}\,\underbrace{\sigma_{\mathsf{x}}^{\overset{\square}{f}(0)}\sigma_{\mathsf{z}}^{n}}\right]\\
&= \frac{1}{2}\left[\mathbb{1} + \lambda_{mn}\,(-1)^{n\overset{\square}{f}(0)+m\overset{\square}{f}(1)}\, i^{mn}\sigma_{\mathsf{x}}^{\overset{\square}{f}(0)}\,\underbrace{\sigma_{\mathsf{x}}^{m}\sigma_{\mathsf{x}}^{\overset{\square}{f}(0)}}\,\sigma_{\mathsf{z}}^{n}\right]\\
&= \frac{1}{2}\left[\mathbb{1} + \lambda_{mn}\,(-1)^{n\overset{\square}{f}(0)+m\overset{\square}{f}(1)}\, i^{mn}\sigma_{\mathsf{x}}^{\overset{\square}{f}(0)}\,\underbrace{\sigma_{\mathsf{x}}^{\overset{\square}{f}(0)}\sigma_{\mathsf{x}}^{m}}\,\sigma_{\mathsf{z}}^{n}\right]\\
&= \frac{1}{2}\left[\mathbb{1} + \lambda_{mn}\,(-1)^{n\overset{\square}{f}(0)+m\overset{\square}{f}(1)}\, i^{mn}\,\underbrace{\sigma_{\mathsf{x}}^{\overset{\square}{f}(0)}\sigma_{\mathsf{x}}^{\overset{\square}{f}(0)}}\,\sigma_{\mathsf{x}}^{m}\sigma_{\mathsf{z}}^{n}\right]\\
&= \frac{1}{2}\left[\mathbb{1} + \lambda_{mn}\,(-1)^{n\overset{\square}{f}(0)+m\overset{\square}{f}(1)}\, i^{mn}\,\underbrace{\mathbb{1}}\,\sigma_{\mathsf{x}}^{m}\sigma_{\mathsf{z}}^{n}\right]\\
&= \frac{1}{2}\left[\mathbb{1} + \lambda_{mn}\,(-1)^{n\overset{\square}{f}(0)+m\overset{\square}{f}(1)}\, i^{mn}\sigma_{\mathsf{x}}^{m}\sigma_{\mathsf{z}}^{n}\right] \qquad (11.2)\\
\rho_{\mathsf{B}} &= \frac{1}{2}\left[\mathbb{1} + \lambda_{mn}\,(-1)^{\mathcal{N}_{\mathsf{B}}}\, i^{mn}\sigma_{\mathsf{x}}^{m}\sigma_{\mathsf{z}}^{n}\right] \qquad (11.3)
\end{aligned}
$$

At this point there is no memory of $\overset{\square}{f}(0)$ & $\overset{\square}{f}(1)$ and they are lost.

11.5 Density Operator Undoing is Non-Deterministic

Measurement now takes the definite density matrix ρ_{B} from (11.3), as input, along with its orthogonality index $\mathcal{N}_{\mathsf{B}}$. From that, Measurement attempts to compute polarisation information from the density matrix's history. That means working backward through the reversal of the Blackbox-Processes (11.1)→(11.3).

Up to this point, everything has been deterministic. But the first step into the reverse process is not. In the step (11.4)→(11.5), definite values for *both* $\overset{\square}{f}(0)$ and $\overset{\square}{f}(1)$ cannot be recovered because the step (11.2)→(11.3) is not reversible.

Throughout the whole of these density operator processes, the step (11.4)→(11.5) is the extent of the anomaly; but its effect trickles through the rest of remaining processes (11.5)→(11.6).

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