3-space left|right handedness illustrating ambiguity, 28 never expressed by the algebra, 14, 132logic distinct from function space, 115 3-valued logic, viii, 19 3-way orthogonality, 46 $a \ posteriori$ consequence of complementarity, 83 field of complex scalars, 44 unitary Hermicity, ix a priori meaning of, 51 Principles, Axioms or Postulates, 4 status of basis vectors in 3-space, 115 imaginary unit in standard theory, 44 textbook unitary Hermicity, 83 unitary Hermicity contradiction of, 63 is mistaken, ix must be re-thought, 109 accidental coincidence and existential quantifiers, 117 self-reference, 23 uncausedness, 113 in Elementary Algebra, 17 the free particle, 93 the Pauli system, 104, 105 of $\eta = \pm i$, 72 of non-contradiction, 89 ambiguity, see also referential ambiguity

arising out of orthogonality, 132 conveyed in commutators, 80 resolved only through ingress of outside information, viii, 69 suppression of, in standard theory, x anti-diagonal, see diametric anticommuting operators and 3-way orthogonal matrices, 46 definition of, 15 have zero trace, 46 Aspect, Alain, 5 assumed as opposed to provable, 20 hypothesised coincidence, 93, 104, 105 simultaneity, 89 assumptions inadvertent, 14 of standard theory, 111 unnoticed, x unwittingly made, 110 Axioms, of formal systems, 51 Banach space $L^1, 85-87$ complete normed space, 85 existence of integrals, 92 loss of closure on \mathbb{R} , 119 rational, and without inner product, 30 triplet of spaces, 103 Baylis, WE, 65 Bell, John, 5 Birkhoff, Garrett mainstream quantum logics, 19 Bloch sphere, 39 handedness reflection, in the, 72 new referential ambiguity, in, 29

Boolean functions of a binary argument, 22 information and the density operator, 75-80 and the orthogonality index \mathcal{N}_{B} , 75 feeding into the density matrix, 39 meaning of, for Pauli processes, 75-80 pairs (0,1), (1,0), (1,1), 37corresponding involutory matrices, 102linked to Pauli operators $\sigma_z, \sigma_x, \sigma_y, 37$ truly isomorphic to involutory c, a, b; logically improve on Pauli, 63-65 propositions, and logical independence, 22 representation of the Vienna Experiments, 35, 37–42 system, of Tomasz Paterek et al, 19-22 in agreement with Reichenbach, 19 not subject to Gödel's Theorem, 43 bound variables: change of variable, 54 Brukner, Časlav, 35 Buridan's Ass 'paradox', 32, 41 Canonical Commutation Relation and homogeneity of space, 84-90 broken by non-unitary scalings, 84 Cantor's Diagonal Argument, 136, 138-141 Theorem, 28, 116, 136 cardinality, 138, 146 Category Theory, 136 cause and non-prevention, 127-128 chains of, ix concepts of, 4–7 in the reductionist sense, 4 in the temporal sense, 4 cause & effect, traditions in Physics, 3, 127chance coincidence, 23, 113, 117, 131 experiments of, 5 circularity, see self-reference, also self-referential classical logic, 18, 29 philosophy, 127 physics, 5, 127

randomness

is deterministic, 5

stems from ignorance of conditions, 5

world, 3, 8, 123 closure in the involutory algebra c, a, b, 64 on \mathbb{R} , lost for orthog' Banach spaces, 119 over scalars in Elementary Algebra, 18 under the Field Axioms, 52 coin-tossing experiments, 5 collapse, 32, 35, 41 caused by breaking self-reference, 113 insight into the workings of, 41 commutative diagram, 141, 143, 146, 147 commutators and Lie algberas, 14 mixed state, irreversible process, 77, 80 pure state, reversible process, 80 identical, see identical commutators complementarity and consequent unitarity, 83, 90 Reichenbach's logic, 19 self-consistency of, 84, 91, 110 the implied imaginary unit, 13 arising from accidental coincidence, 91, 93, 104, 105 conveys referential ambiguity, 29 implied in mixed states, 13 not demanded by pure states, 91, 102 complementary spaces accounting for whole information, 11 and multi-loop circularity, 31 fixed by consistent self-reference, 117 treat combined as a single system, 14 variables in the density operator, 31 Completeness Theorem, see Gödel's Completeness Theorem (1929) complex, see also imaginary unit field of scalars \mathbb{C} , 18 interpretation used in truth-tables, 58 scalars; question of *a priori* versus a posteriori status, 44 configuration of hardware, as information conveyed through experiments, 35-42, 76, 102consistency and semantic information, 11 and Soundness & Completeness and provability, 56 as distinct from implication, x as Principle defining the generic Theory, 10

the *Theory* of Elementary Algebra, 114 dependency of, on the imaginary unit, 46, 106 on unitarity, 110 maintained by fixed-points, 28 with the Field Axioms, 53 consistent complementarity, 84, 89, 91 contradiction, denying textbook a priori unitary Hermicity, 13, 29, 83 crossover-loop circularity, 92 decisions made by Buridan's Ass, 32, 41 made in photon experiments, 6, 129 density matrix algebraic processes, detail of, 76-80 as carrier of Boolean information (representing orthogonality), 38–42 history, compared to planet history, 128 insight into commutator processes, 69 interaction with measurement, 125 referential ambiguity inherent in, 31-32 the ingression of, 75-80 symmetry in photon histories, 32 density operator, see density matrix detectors, of transmitted or reflected photons, 5-6 deterministic evolution of the density operator, 31, 38-41, 77, 80, 125 experiments of chance, 5 motion of planets, 8, 127, 128 diagonal and cartesian square, 144 Cantor's, 138-140 function \triangle general definition, 144 $\triangle_B, 141$ $\triangle_{\mathbb{N}}, 143$ diametric, of Cantor's diagonal, 138, 139 dice-throwing experiments, 5 Dirac delta function defined as, 95 disagreement between fields, 58 between models, 56 within a truth table, 54, 59 discrete decisions, 6, 129 objects, 6, 129 photons, 5 displacement, see translation

displacement parameter ϵ . 85 Dominated Convergence Theorem, 95, 96 dummy variable of integration, 12, 92 eigenformulae (formally quantified) for free particle, pure state superpositions, 11-12, 91 for position operator \mathbf{x} , 86 the trivial, 88 eigenfunctions χ_k of wavenumber operator k, 88 $\psi_{\rm x}$ of position operator **x**, 87 $f_{\rm x}$ of position operator **x**, 86 eigensolutions $Af_i = \lambda_i f_i$ identity mappings, 116 of flexible involutory matrices, 73 eigenstates information in density operator, 31 photons prepared as, 36 eigenvalues as fixed-points, 28 of flexible involutory matrices, 73 of position x, 86 postulated realness of, 110 realness guaranteed by complementarity, 84 eigenvector and single loop circularity, 116 position, 85 Einstein, Podolsky & Rosen paradox, 19 Elementary Algebra as environment for 3-valued logic, 43-44 formal system, 51–55 information entering Quantum Mathematics, 26, 27 information underlying logically active linear algebra, 30–31 logical foundation to Mathematical Physics, 49–50 machinery of linear algebra, 45, 115 machinery of uncaused self-reference, 113theory of existence, 114 furnishes the mathematical structures of Quantum Theory, 114 independence test, truth-tables, 58 introduction to, 17-18 logically independent statements examples of, 53-55 infinite number of, 21, 43-44 epistemology and newly created indefiniteness, 113

explains indefiniteness, viii of a perfect disc, 124 of a perfect square, 124 of perfect symmetry, x, 123–125 of prepared states (Reichenbach), 19 relating to Boolean propositions, 35 unknowable, position in a void, 123 EPR paradox, see Einstein, Podolsky & Rosen paradox Euclid's Theorem on Primes, 137, 142-144 evidence, see also Contents pages, viii about the, 25 evolution, in density matrix processes and ingress of information, 69 and irreversibility, 41 from Measurement's viewpoint, 79 excluded middle, 56-58, 128 existence of scalars and the Field Axioms, 53–55 asserted by formulae, 53, 92, 103, 114 theory of, x, 114 existential quantification, and propositions of accident, 117 quantum theory, x experiment hardware and density matrix, 125 configuration, 35, 76 Preparation, Blackbox and Measurement, 36 faithful isomorphic representation and a priori unitarity, 13 difference in pure states and mixed, 68.69 logic of mixed states, 101 matrices for pure states, 71–73 Pauli algebra *required* by mixed states only, 28 unitary on-off switching, 28-30 whole information, 86 for Canonical Commutation Relation and homogeneity of space, 84-90 for pure and mixed states, 13, 111

in the density matrix, 76 must be insisted upon, 64

is non-unitary, 91, 102

machines, capable of, 116, 131

disagrees with textbook theory, 29

of pure states

stability, 102

feedback

Feynman, Richard, glass plate experiment, 5 Field Axioms, 52-53 and 3-valued logic, 43 derivability of pure states, 92 logical independence, 43-46, 49 logically independent machinery, 45 - 46Model Theory, 56–59 their formal system, 52–58 are subject to Gödel's Theorem, 43 continual operation of, 114 examples of proof, from, 55 propositions, under, 53-55 in relation to circular self-reference, 92 Elementary Algebra, 18 Quantum Mathematics, 26, 27 structures in Linear Algebra, 30 semantics of, 11 fields of scalars as objects consistent with Field Axioms, 11 in Elementary Algebra, 114 structures \mathbb{Q} , \mathbb{R} , and \mathbb{C} , 18 First Incompleteness Theorem, see Gödel's Incompleteness Theorem (1930) fixed-point absence of, 137-138, 142-144, 146-148 and Cantor's theorem, 116 and self-reference, 136 as eigenvalue, 28 imaginary unit, 45 in a map, 28 in Fourier Inverse Theorem, 94 mapping, 111, 131 Fixed-Point Theorem, see Lawvere, F William fixing symmetry and Cantor's theorem, 28 unitarity, 111 flexible 3-space, 22 matrices, 71 Form Invariance applied to the position eigenformula, 86 concept from relativity, 84 formal system, 9 Boolean propositions, treated as, 20-22 Elementary Algebra, treated as, 18, 51 - 55

formal theory. see formal system Fourier Inverse Theorem formal logical approach to, 91 Fourier transform and inverse crossover-loop, 116 Frederic Schuller, 95 freedom allowed by logical independence, 8 in *actual* quantum systems, x lost at measurement, 6, 129 permitted by Quantum Mathematics, x, 9 to permit, without cause, 4 to transmit, or reflect (photons), 6, 129 Fubini, 96 function space, logic distinct from geometric space, 115 geometric space logic distinct form function space, 115 glass sheet experiment, 5 Gödel, Kurt, vi, viii, x, 4, 28, 43-45, 69 theorems of, x Gödel's Completeness Theorem (1929), 44, 45, 53 - 59Incompleteness Theorem (1930) and Classical paradoxes, 135, 136, 148 and existence of logical independence, 21announcement of, vi Boolean system, not subject to, 43 exemplar of, 28 Field Axioms are subject to, 43 information from outside, 69 true but unprovable statements, viii, 4 gravitation as 'reductionist' law of motion, 4 motion has definite history, 32 group additive. 52 corresponding Lie algebra, 85 epistemology of, 124-125 general linear $GL(\mathbb{F})$, 85 Lie, 85 multiplicative, 52 orthogonal, 14, 15 relation representing homogeneity, 85, 88 symmetry, 109 theories covered by blanket unitarity, 83 unitary, 15

handedness. see left right handedness Hardegree, Garv M, 19 hardware, see experiment hardware Hermicity, as consequence of unitarity, 15, 110 Hermitian conjugate, 15 observables, 83, 109, 110 operators, 30, 103 hidden variables, 5 Hilbert space as subspace of Banach space, 85 history, of state preparation ambiguation of, 125 bifurcation in, 125 compared to planetary motion, 128 lost at measurement, 75-80 no definite memory of, 41 non-uniqueness of, 31-32 homogeneity of space, 83-90 unitary subsymmetry, of, 85 Huschilt, J, 65 hypothesised coincidental amplitudes in the free-particle, 93 Pauli system, 104 scalings in the Canonical Commutation Relation, 89–90 identical commutators, of symmetric product-pairs, 80 identity mapping, 27, 116-117 ignorance, see classical randomness imaginary scale factor needed for self-consistent selfreference, 117 unit as visual marker for circularity, 31 crucial existence of, vi

% inherent in multi-loop circularity, 31 logical independence of, $51\mathchar`-58$

logically independent role of, 44–50

machinery necessitating, 45–50 necessary versus possible, existence of, 11–13

replacement by η for pure states, 12, 65, 71, 91–107 self-referential, reliance upon, 103

well-known logical independence, of, 6 Incompleteness Theorem, *see* Gödel's Incompleteness Theorem (1930)

indefiniteness, see indeterminacy

indeterminacy and Reichenbach's 3-valued logic, 19 as the association of uncausedness and indefiniteness, viii, 4, 26, 27, 32, 38, 132as the gap between pure and mixed states, 26, 27, 30 experimental example of, 5 hidden within Elementary Algebra, 17 indefiniteness of, 28, 32, 69, 75-80, 123 - 125is not irreducible, 113 mathematical processes, of, 25 matrix representation of, 71–73 necessary versus possible information, 11 originates in Quantum Mathematics, 9 question of irreducibility, 3, 113 referential ambiguity of, 75-80 relation to quantum randomness, 5 the role of unitarity, 109–111 uncausedness of, 113-119, 127-128 ingression after the Field Axioms, 26, 27 into the pure state spectrum, 110 not subject to any conservation law, 110 of logically independent information, 23 orthogonality, 67-69 unitarity, 63 through the Blackbox, 38 inner product, see also scalar product and Banach space, 85 homogeneity, 87 self-reference, 30, 103 self-referential definition of, 117 simultaneity, 30 intuitive unintuitive experiments, vii invariance, of position operator under homogeneity, 86 probability, 109 probability amplitude, 83 scalars, as first rank tensors, 16 trace, of similarity transformations, 47 involutarity in the Vienna Experiments, 69 of the Pauli matrices, 72 involutory cancellations, 80 environment of Vienna Experiment pure states, 110 information, 66-68

in Quantum Mathematics, 26 matrices, 71 irreducible is quantum indeterminacy?, 3 quantum indeterminacy is not, 113 irreversibility in density matrix processes, 41-42, 77 isomorphic step-transition, across pure mixed states, 71, 102, 111 isomorphism between indeterminacy and logic of maths, viii by Principle, must be entirely faithful, 64 false, between Boolean pairs and Pauli, 64 pure and mixed state operators, 111 Vienna Experiments and Pauli algebra, 63-65 $\sigma_{\rm x} | \sigma_{\rm y}$ ambiguity, with + |-, left|right handedness, 80 with involutory algebra c, a, b, 69 logic of epistemology of states, 19 Pauli algebra, wrongly assumed, 63 Jiansu Wei, 65 Karimi & Salehi, 137, 140, 142 Kochen and Specker, 5 Laws of Motion, 4, 127 Physics, x Lawvere, F William, Fixed Point Theorem, 136, 148 Lebesgue integrable, 96 Lebesgue integrable functions, 87 left|right handedness added to a void Universe, 123 ambiguity broken, 132 under reflection. 22 as new referential ambiguity, 29 in the Bloch sphere, 32 not specified in the algebra, 69 Pauli products ambiguity, 80 reflection in the Bloch sphere, 72 specified by outside information, 28 stemming from new orthogonality, 132 unrecognised by scalars, x Liar paradox, 136 Lie algebra, 15, 64-66, 88 su(2), 65

representing homogeneity, locally, 83 - 90Lie group, representing homogeneity, 83 - 90logical dependence and cause, 127 as syntactic information, 54 effect of, in the Vienna Experiments, 38.40followed through experiments, 38 in Boolean propositions, 22 in pure states, 125 linked to predictability, 35 of rationals, upon the Field Axioms, 18 under Soundness & Completeness, 56 logical independence, see Contents pages loops of circularity, 113-119 causal loops, ix crossover-loop, and the Fourier transform, 92 multi-loops, and mixed states, 28 orthogonality, 30-31 unitary symmetry, 28, 111 single-loops, and pure states, 28 Łukasiewicz, Jan, 19 machinery, see also mechanism, viii, 130 affecting uncaused indeterminate phenomena, 113 explaining uncausedness and indefiniteness, 26 logically independent, 10 necessitating the imaginary unit, 45-49 of indeterminacy, x, 4, 64, 129 of information conservation, and loss, 76self-referential, viii that breaks symmetries, 125 magnetic field, and caused collapse, 32, 125mapping Banach spaces into themselves, 87 Boolean pairs to Pauli operators, 37 fixed-point, 111 identity, 27, 116–117 of the Vienna Team, 63 self-referential, 28, 45 to a contradiction, 142 mathematical discipline, 9 information, 5 undecidability, x, 44

and Vienna Experiments, 135 Mathematical Logic, 9 mathematician's inadvertent assumptions not asserted in the maths, x Matrix Mechanics accepting unitary on-off switching, 111 faithful for pure and mixed states, 71 Vienna implications for, 35 measurement and ambiguity in the density operator, 125 Buridan's Ass, 32 epistemology of perfect symmetry, 123 - 125faithful isomorphism, 111 inaccessible lost history, 75-80 quantum randomness, 35-42 causes unitary symmetry breaking, 32 of photon polarisation, 10 prior to, 10 state immediately before, 19 Measurement Problem, The, viii, 3, 6, 129 mechanism, of, see also machinery freedom, permitting indeterminacy, 6, 129indeterminacy, viii, 4 and the imaginary unit, 83 ingress of the imaginary unit, 44-49 irresistible progress of the measurement decision, 41 measurement's loss of freedom, 6, 129 self-referential circularity, 29 in free particle Wave Mechanics, 91 in Matrix Mechanics, 103-107 memory, and density operator, 76 irreversibility, 41 lost input $\overline{f}(0)$ & $\overline{f}(1)$, 78 orthogonality index \mathcal{N}_{B} , 40 mixed states and necessary unitarity, 12 referential ambiguity, 31-32 semantic interpretation, 10 the orthogonality index \mathcal{N}_{B} , 39 unitary switch-on, 29 ingression of orthogonality, 67-69 logical transition, 29–30 mathematical comparison of, 11-13 meaning of, 10 not bijective with history, 125 randomness of, 10

unitarity already in implied in, 29 Modal Logic or logical independence is needed, 29 possible \diamond , or necessary \Box , 37 possible necessary, 13, 29 unitarity, in the sense of, 75 Model Theory, 56–58 momentum space and the Canonical Commutation Relation, 84, 90 interdependence with position space, 11, 14.92 necessary, see possible necessary neg, the function, 143-144, 146-148 Newton interference. 6 Laws of Motion, 4 non-ambiguity in parallel polarisation experiments, 125 pure states, 113 non-classical logic of Reichenbach, 19, 43 non-deterministic, "undoing" of density operator, 78-80 non-prevention in Elementary Algebra, 114 in uncaused effects, 127-128 of self-referential circularity, 32 non-unitary homogeneity of space, 84 outlawed in standard theory, 29 pure state algebra c, a, b, 63-69 Pauli system matrices, 71-73 representation of free particle, 91-92 representation of Pauli system, 102 - 103to unitary on-off switching, 110-111 norm L^1 space, 87 L^2 space, 87 normalisable, 87 of a Banach space, 85 of a Hilbert space, 87

one-one and onto correspondence failure of, within Boolean-Pauli relationship, 63 density operator history, 125 geometry-scalar relation, 77 preparation processes, 80 ontological cause, 4 ontology

complex scalars, as, 44 is non-unitary, 90, 110 of standard theory, ix orthogonal anticommuting definition for, 14–15 function and geometric spaces, logically different, 115 group, 14, 15 information in the Pauli algebra, 66-69 inner product, replacement by self-reference, 30 polynomials, 115 rotations implying imaginary unit, 45 vectors logical independence between, 22 orthogonality 3-way, 46 and logical comparison of, for 3-space against function spaces, 115 the implied imaginary unit, 45-46, 49 by chance accident, 117 denial of, and caused collapse, 32 entering step transition matrices, 73 evolution through experiments, 76 implied existence of imaginary unit, 31 imposed though self-reference, 101-107 ingression into mixed states, 67–69 introduced into Elementary Algebra, 31 self-referential definition of, 117-119 within Elementary Algebra, 50 orthogonality index \mathcal{N}_{B} as used by Paterek et al, 39-42 definition of, 39 interaction with density operator, 75 - 80no memory of how generated, 40 paradox Buridan's Ass, 32 Classical, 135, 136, 140, 148 EPR, 19 Liar, 136 orthogonal rotations mapping vectors parallel to themselves, 45 Russell, 135, 136, 148 Pauli algebra as unitary restriction on states, 110 derivation from six statements, 67 the involutory algebra c, a, b, 67, 68 information content of, 65-69 is a Lie algebra vector space, 65

meaning for, implicit in Boolean information, 75 new insight, for, 69 only mixed states require this, 28 processes in the density matrix, 75-80 Pauli matrices, 15 as result of self-reference, 102-107 Pauli operators are anticommuting, 15 basis vectors for a Lie algebra, 15 not isomorphic with Vienna Experiments, 63–65 interacting with orthogonality index $\mathcal{N}_{\mathsf{B}}, 76$ product-sequence, at measurement, 80 specifying Boolean propositions, 38 Pauli system evidence taken from, 25 is left right ambiguous. 32 logically independent unitarity, in, 101 - 107Pauli transformations, in the Vienna Experiments, 36 perfect symmetry and care needed by mathematicians, x unitary mixed states, 32 creation of, introduces referential ambiguity, 29 epistemology of, 123-125 left|right handedness, example of, 28 referentially ambiguous epistemology explains indefiniteness, viii *permitted* physics, as opposed to *caused*, 4, 9, 27, 32, 72, 102, 103, 110, 111, 116 photon, 10 epistemology of states, 19 transmission reflection experiment, 5–6, 129photon polarisation density operator, 75-80 matrices for pure states, 71–73 Vienna Experiments, 10, 35 without complementarity, 102 physical information, 5 planets, 8, 32, 127, 128 and apples, 4 polarisation, see photon polarisation polarisers and Measurement epistemology, 80 as symmetry breakers, 32, 125 orientation and alignment of, 76

used, in the Vienna Experiments, 36 possible necessary information in a single theory, 12 Quantum Mathematics, 11 unitarity and quantum randomness, 29 as Modal Logic, 29, 75 Postulate as a priori cause, 3-7, 127, 130 unitary|Hermitian breaking free of, 13 evidence for revision of, 25 experimental contradiction of, ix, 63 - 65mathematical redundancy of, ix revised approach to, 111 suppresses logic of indeterminacy, 43 predeterminated properties, see hidden variables predictability and orthogonality, 69 randomness, representation of, 71, 111 in context of probability, 6 randomness, 5 predictable experiments, and outcomes linked with logical dependence, 35 of pure states, 10 redundant unitarity, 28 Preparation in Vienna Experiments, 36 preparation history and density operator evolution, 31 inaccessible and lost, 125 reversal of, is not deterministic, 80 prepared states conveying referential ambiguity, 29 epistemology of, 19 mixed, convey referential ambiguity, 31 Principle, see Postulate prior-to-measurement epistemology, 35 probability, 6 amplitude and complementarity, 84 in standard theory, 44 is complex, 11, 110 need for unitary theory, 83 density, 76 epistemology of, 124 information, 76 invariance of, 109

product-pairs, 80 product-sequence of Pauli operators, 80 $\sigma_{\rm x}\sigma_{\rm z}\sigma_{\rm x}^{\dagger}$ or $\sigma_{\rm y}\sigma_{\rm z}\sigma_{\rm y}^{\dagger}$, 40, 76 transformations, 29 as machines, 116 chained, 131 circular, 111 propagation, of orthogonality index \mathcal{N}_{B} , 40 photons along the Y axis, 36 propositions and truth-tables, 58 Boolean, 35, 37-38, 42 in a formal system, 20–22 in Elementary algebra, 51-55, 58 of chance accident, 117 under Soundness & Completeness, 56 Putnam, Hilary, 19

quantifier for-all \forall . 51 there-exists \exists , 51 logic advantages for Quantum Theory, x Wave Mechanics example, 13 rule for formula substitution, 54 quantum indeterminacy, see indeterminacy Quantum Mathematics, 9–15 and *a posteriori* unitarity, 83 and the imaginary unit necessity for, 83 role of, 44 root origin of, 44 as evidence, 110 as whole information, 76 circularity and referential ambiguity within, 27 definition of, 9 flowchart of information, 26 freedoms imposed by, x importance of Elementary Algebra, 17 in the hierarchy of theories, 49 indeterminacy concerns, 9 logical independence within, 35, 43, 44 logical standing of unitarity, 109 possible versus necessary interpretation, 11 self-referential information in, 113 treated as a formal system, 9 unitarity imposed by self-reference, 119 unitary Hermitian terminology, 15 Quantum Postulate, see Postulate

quantum randomness. 5 & logical independence, 35-41 concepts of, 3 effect neither caused nor prevented, 128 found not irreducible, viii in contrast to classical randomness, 5 in mixed states only, 10 involves logical independence, 5 originates in mathematical processes, 5 orthogonality is predictor of, 69 relation to indeterminacy, 5 theoretical improvement on Pauli, 64 Vienna Experiments, link with logical independence, 6 unitarity, 28 randomness, see quantum randomness, also classical randomness rational field (infinite) as model of the Field Axioms, 58 the smallest, 52 linear spaces, 30 numbers, and closure, 18 Elementary Algebra, 52 scalars as consequence of Axioms, 18, 46 in context of derivability, 92 variables interpreted as, 11, 54 reductionist cause, 4 redundant added axiom, 12 axiomatic unitarity, 86 information absence of, in ideal axiom systems, 21 absence of, in minimal axiom set, 131 in proofs, 20 Quantum Postulate, ix unitarity, 13 blocks logic of mixed states, 29 shown to be, 28 unitarity |Hermicity, 84 Unitary Postulate, 110 referential ambiguity, see also ambiguity and scalar algebraic inadequacy, in conveying geometry information, 77 asepistemology of perfect symmetry, 123 - 125root of indefiniteness, 75 conveyed in the density operator, 31 created by new symmetry, 29, 72

evolves in the density matrix, 76 explains indefiniteness, 27 in (unitary symmetric) Bloch sphere, 32 left|right handedness, example of, 28 presence in mixed states, 31 regularisation by Gaussian, 95–97 Reichenbach, Hans 3-valued logic, viii, 19-20 'true' is not the same as 'not false'. 19 agreement with Paterek's Boolean maths, 19 exerted by Elementary Algebra, 18, 43 not opposed to Birkhoff and von Neumann, 19 supported by Hardegree, 19 supported by Putnam, 19 replicated formulae, viii, 31 representable (diametric), 141–143, 147 representation, see also faithful isomorphic representation by formal system, 9 by momentum and position spaces, combined, 11 faithful to the logic of experiments, 111 historical status of unitary groups, 83 in standard textbook theory, 110 matrices of anticommuting operators, 47 indeterminate information, 71-73 must be *entirely* isomorphic, 64 of indeterminacy, 30, 44 pure to mixed state, logical jump, 12 Vienna Experiments, assumes independent orthogonality, 68 using Boolean propositions, 37–38 Rosinger, Elemér, 50, 117-118 $\mathsf{S}(\epsilon)$ one-parameter subgroup of $\mathsf{GL}(\mathbb{F})$, 85.88 scalar product, see also inner product and logical comparison for 3-space against function spaces, 115 scalars and Elementary Algebra, 17-18 arrays of, in mathematical machines, 45 logical significance of, 11 semantic interpretation of, 10 under the Field Axioms, 53-55scale factor arbitrary n, 12, 64, 72, 88, 91, 103 imaginary, maintaining self-consistency, 46, 84, 117

scalings and fixed-points etc, 84, 102, 117 relating wavenumber and position spaces, 90 self-adjoint, see Hermitian self-consistent 3-way orthogonality and the imaginary scale factor, 46 complementarity and accidental coincidence, 91 and the imaginary scale factor, 84 by way of elimination, 110 self-reference and caused collapse, 113 orthogonal transformations, 50 self-inconsistence, 117 as machinery of uncausedness, viii capable of Linear Algebra, 30 cyclic and anti-cyclic, 103 evidence for, viii linked to theorems of Kurt Gödel, x logically active linear algebra, 30-31 need for ingress of information, 23 within 'The Universe'?, ix the free particle mixed state, 91 the Pauli matrices, 101–107 unitary on-off switching, 29 self-referential answer for Philosophy of Physics, 113 definition of orthogonality, 50, 117 machinery of fixed unitary symmetry, 131 fixed-point imaginary unit, 45, 107 fixed-points, 28 Hermitian operators, 30 Hilbert spaces, 30 orthogonality, 101-107 stable entities, 27, 28 uncausedness, 27, 32, 113-119 unitarity, 106-107, 119 mapping, 28, 45 multi-loop orthogonality, 31 need for ingress of information, 23 operations in Quantum Mathematics, 113 - 119semantic interpretation, 10-12 and the Completeness Theorem, 54 and the Soundness Theorem, 54 as rational, real or complex, 54 in a truth-table, 58–59

sentence, meaning of, 54 separable functions, 87 sequence, see product-sequence simulation of 'The Universe', x simultaneitv and ingress of the imaginary unit, 45–50 logically active Linear Algebra, 30-31 in Elementary Algebra, 27 simultaneous coexistence, 115 equations can induct independence, 23 validity, 89 Soundness Theorem, 44, 45, 54, 56 Soundness & Completeness Theorems, 53, 56 - 59span, of a Banach space, 115 Lie algebra, 14 spontaneous operation of Field Axioms, 45 square array, 138-140 stability, 17 of eigenvalues, eigenvectors, unitary symmetries, 131 of identity transformations, 27, 111 stable, persistent circulation, 103 feedback, 102 quantum states, 113 systems, 26 standard textbook quantum theory, ix, x, 12, 13, 25, 29, 44, 87, 91, 109, 111 step-transition between pure and mixed states, 29, 68, 73, 91, 101, 102, 111 structure in Elementary Algebra, expressing Linear Algebra, 115 logic of indeterminacy, 114 logical. in Quantum Mathematics, viii of self-reference in Quantum Mathematics, 113 replacing definitions, 30, 49 of a formal system, 20 representing mixed states, 101 structures (mathematical) as models in Model Theory, 56-59 infinite fields \mathbb{Q} , \mathbb{R} , and \mathbb{C} , 18 substitution, involving quantifiers \exists , \forall , 54 superfluous, see redundant switching, see unitary on-off switching

symmetries broken by point-marking, 124 epistemology of, 124–125 syntactical information, 10 syntax, 10 Szangolies, Jochen, 137 tensor, 16 orthogonal, 101 test for logical independence, 53–54, 58 textbook theory, see also standard textbook theory, ix, x, 13, 14, 84, 85, 109Theorems Cantor's, 28, 116 Completeness (1929), 44, 45 Fourier Inverse, 91 Gödel's First Incompleteness (1930), vi, viii, 21, 43, 69 Kochen and Specker, 5 Soundness, 44, 45 Soundness & Completeness, 53–59 theorems, and apparent theorems, 23 Theory, meaning of, in this book, 10, 114 thought experiment concerning epistemology of unmarked symmetries, 124 self-reference in The Universe, viii, ix Tomasz Paterek et al, viii, 5, 19, 35 Tonelli, 96 trace zero, and anticommutation, 46 traditions, of Applied Mathematics, 9, 11, 12, 49, 114 Physics, 3, 130 transformations as stable machines, 116-117 chained sequences of, 131 complying with Form Invariance, 84 homogeneity, 85-89 Fourier logic in Wave Mechanics, 91 going on within 3-way orthogonality, 46-49 machines, 45 machines in indeterminacy, 131–132 polarisation experiments, 36 the density operator, 75-80 the Pauli step transition, 102–107 seen as simultaneous equations, 23, 115 typical of Lie algebras, 15 translation and the homogeneity of space, 84–87

passive, 86 transmission reflection experiment, 5 true false disagreement, 54, 58 truth-table, 58-59 turnstile symbols, derives \vdash , models \models , 51 uncaused effects, ix and indeterminacy, 127-128 indeterminate phenomena, 113 unprevented phenomena, 8 uncausedness, see indeterminacy undecidability, see mathematical undecidability undecidable statements, x undoing, density operator processes, 41, 77 - 80unitarity a posteriori status of, 83-84 and Canonical Commutation Relation, 84 - 90caused collapse, 32 homogeneity of space, 84–90 indefiniteness, 32 uncausedness of indeterminacy, 32 by Quantum Postulate contradicts Vienna Experiments, 63 - 65is irrelevant and obstructive, 29 is mathematically redundant, ix is a posteriori, not a priori, ix necessary versus possible status of, 12 - 13revised approach to, 109-111 unitary definition of, 15

left|right handedness of, not specifiable from within, 28 logical independence in the free particle, 91 in the Pauli system, 101-107 on-off switching, 28-29, 111 switch-off forces collapse, 32 symmetry, fixed by multi-loop self-reference, 28 Unitary Postulate, see Postulate unitary|Hermitian Postulate, see Postulate unitary|Hermitian, terminology, 15 unprevented phenomena, 8 self-referential circularity, 29 van Zyl, Gusti, 50, 117-118 von Neumann, John density operator, 31 mainstream quantum logics, 19 wave continuum, expresses probability, not decision, 6, 129 Wave Mechanics and homogeneity of space, 83-90 momentum versus position space, 14 wave plate, 36 whole information and the density matrix, 76 of Hilbert space, logical structure, 30 of homogeneity, 86 of the free particle system, 11 principles for the conveyance of, 13-14 Yanofsky, Noson, 136, 137, 145

Zeilinger, Anton, 35